Chapter - Atoms

Topic-1: Atomic Structure and Rutherford's Nuclear Model

1 MCQs with One Correct Answer

1. An alpha particle of energy 5 MeV is scattered through 180° by a fixed uranium nucleus. The distance of closest approach is of the order of **[1981- 2 Marks]**

(a) 1 Å (b) 10^{-10} cm
(c) 10^{-12}cm (d) 10^{-15} cm

Topic-2: Bohr's Model and the Spectra of the Hydrogen Atom

1 MCQs with One Correct Answer

1. A metal target with atomic number Z = 46 is bombarded with a high energy electron beam. The emission of X-rays from the target is analyzed. The ratio r of the wavelengths of the K_α -line and the cut-off is found to be $r = 2$. If the same electron beam bombards another metal target with Z = 41, the value of r will be **[Adv. 2024]**
(a) 2.53 (b) 1.27 (c) 2.24 (d) 1.58

2. Ina hydrogen like atom electron make transition from an energy level with quantum number n to another with quantum number (n – 1). If n>>1, the frequency of radiation emitted is proportional to : **[2012]**
(a) $\frac{1}{n}$ (b) $\frac{1}{n^2}$ (c) $\frac{1}{n^{3/2}}$ (d) $\frac{1}{n^3}$

3. The wavelength of the first spectral line in the Balmer series of hydrogen atom is 6561 A°. The wavelength of the second spectral line in the Balmer series of singly-ionized helium atom is **[2011]**
(a) 1215 A° (b) 1640 A° (c) 2430 A° (d) 4687 A°

4. The largest wavelength in the ultraviolet region of the hydrogen spectrum is 122 nm. The smallest wavelength in the infrared region of the hydrogen spectrum (to the nearest integer) is **[2007]**
(a) 802 nm (b) 823 nm (c) 1882 nm (d) 1648 nm

5. A photon collides with a stationary hydrogen atom in ground state inelastically. Energy of the colliding photon is 10.2 eV. After a time interval of the order of micro second another photon collides with same hydrogen atom inelastically with an energy of 15 eV. What will be observed by the detector? **[2005S]**
(a) One photon of energy 10.2 eV and an electron of energy 1.4 eV
(b) 2 photon of energy of 1.4 eV
(c) 2 photon of energy 10.2 eV
(d) One photon of energy 10.2 eV and another photon of 1.4 eV

6. If the atom $_{100}Fm^{257}$ follows the Bohr model and the radius of $_{100}Fm^{257}$ is n times the Bohr radius, then find n. **[2003S]**
(a) 100 (b) 200 (c) 4 (d) 1/4

7. The electric potential between a proton and an electron is given by $V = V_0 \ln \frac{r}{r_0}$, where r_0 is a constant. Assuming Bohr's model to be applicable, write variation of r_n with n, n being the principal quantum number? **[2003S]**
(a) $r_n \propto n$ (b) $r_n \propto 1/n$
(c) $r_n \propto n^2$ (d) $r_n \propto 1/n^2$

8. A Hydrogen atom and a Li^{++} ion are both in the second excited state. If ℓ_H and ℓ_{Li} are their respective electronic angular momenta, and E_H and E_{Li} their respective energies, then **[2002S]**
(a) $\ell_H > \ell_{Li}$ and $|E_H| > |E_{Li}|$
(b) $\ell_H = \ell_{Li}$ and $|E_H| < |E_{Li}|$

(c) $\ell_H = \ell_{Li}$ and $|E_H| > |E_{Li}|$

(d) $\ell_H < \ell_{Li}$ and $|E_H| < |E_{Li}|$

9. The transition from the state $n = 4$ to $n = 3$ in a hydrogen-like atom results in ultraviolet radiation. Infrared radiation will be obtained in the transition **[2001S]**

(a) $2 \to 1$ (b) $3 \to 2$ (c) $4 \to 2$ (d) $5 \to 4$

10. The electron in a hydrogen atom makes a transition from an excited state to the ground state. Which of the following statements is true ? **[2000S]**

(a) Its kinetic energy increases and its potential and total energies decreases.

(b) Its kinetic energy decreases, potential energy increases and its total energy remains the same.

(c) Its kinetic and total energies decrease and its potential energy increases.

(d) Its kinetic, potential and total energies decrease.

11. Imagine an atom made up of a proton and a hypothetical particle of double the mass of the electron but having the same charge as the electron. Apply the Bohr atom model and consider all possible transitions of this hypothetical particle to the first excited level. The longest wavelength photon that will be emitted has wavelength λ (given in terms of the Rydberg constant R for the hydrogen atom) equal to **[2000S]**

(a) $9/(5R)$ (b) $36/(5R)$ (c) $18/(5R)$ (d) $4/R$

12. In hydrogen spectrum the wavelength of H_α line is 656 nm, whereas in the spectrum of a distant galaxy, H_α line wavelength is 706 nm. Estimated speed of the galaxy with respect to earth is, **[1999S - 2 Marks]**

(a) 2×10^8 m/s (b) 2×10^7 m/s

(c) 2×10^6 m/s (d) 2×10^5 m/s

13. As per Bohr model, the minimum energy (in eV) required to remove an electron from the ground state of doubly ionized Li atom ($Z = 3$) is **[1997 - 1 Mark]**

(a) 1.51 (b) 13.6 (c) 40.8 (d) 122.4

14. Consider the spectral line resulting from the transition $n = 2 \to n = 1$ in the atoms and ions given below. The shortest wavelength is produced by **[1983 - 1 Mark]**

(a) Hydrogen atom (b) Deuterium atom

(c) Singly ionized Helium (d) Doubly ionised Lithium

2 Integer Value Answer

15. A Hydrogen–like atom has atomic number Z. Photons emitted in the electronic transitions from level n = 4 to level n = 3 in these atoms are used to perform photoelectric effect experiment on a target metal. The maximum kinetic energy of the photoelectrons generated is 1.95 eV. If the photoelectric threshold wavelength for the target metal is 310 nm, the value of Z is ______. **[Adv. 2023]**
[Given: hc = 1240 eV–nm and Rhc = 13.6 eV, where R is the Rydberg constant, h is the Planck's constant and c is the speed of light in vacuum]

16. Consider a hydrogen-like ionized atom with atomic number Z with a single electron. In the emission spectrum of this atom, the photon emitted in the n = 2 to n = 1 transition has energy 74.8 eV higher than the photon emitted in the n = 3 to n = 2 transition. The ionization energy of the hydrogen atom is 13.6 eV. The value of Z is ______. **[Adv. 2018]**

17. An electron in a hydrogen atom undergoes a transition from an orbit with quantum number n_i to another with quantum number n_f. V_i and V_f are respectively the initial and final potential energies of the electron. If $\frac{V_i}{V_f} = 6.25$, then the smallest possible n_f is **[Adv. 2017]**

18. A hydrogen atom in its ground state is irradiated by light of wavelength 970 Å. Taking $hc/e = 1.237 \times 10^{-6}$ eV m and the ground state energy of hydrogen atom as –13.6 eV, the number of lines present in the emission spectrum is **[Adv. 2016]**

19. An electron is an excited state of Li^{2+} ion has angular momentum $3h/2\pi$. The de Broglie wavelength of the electron in this state is $p\pi a_0$ (where a_0 is the Bohr radius). The value of p is **[Adv. 2015]**

20. Consider a hydrogen atom with its electron in the n^{th} orbital. An electromagnetic radiation of wavelength 90 nm is used to ionize the atom. If the kinetic energy of the ejected electron is 10.4 eV, then the value of n is ($hc = 1242$ eV nm) **[Adv. 2015]**

4 Fill in the Blanks

21. In the Bohr model of the hydrogen atom, the ratio of the kinetic energy to the total energy of the electron in a quantum state n is .... **[1992 - 1 Mark]**

22. The Bohr radius of the fifth electron of phosphorous atom (atomic number = 15) acting as a dopant in silicon (relative dielectric constant = 12) is .... Å **[1991 - 1 Mark]**

6 MCQs with One or More than One Correct Answer

23. A particle of mass m is moving in a circular orbit under the influence of the central force $F(r) = -kr$, corresponding to the potential energy $V(r) = \frac{kr^2}{2}$, where k is a positive force constant and r is the radial distance from the origin. According to the Bohr's quantization rule, the angular momentum of the particle is given by $L = n\hbar$, where $\hbar = \frac{h}{(2\pi)}$, h is the Planck's constant, and n a positive integer. If v and E are the speed and total energy of the particle, respectively, then which of the following expression(s) is(are) correct? **[Adv. 2024]**

(a) $r^2 = n\hbar\sqrt{\frac{1}{mk}}$ (b) $v^2 = n\hbar\sqrt{\frac{k}{m^3}}$

(c) $\frac{L}{mr^2} = \sqrt{\frac{k}{m}}$ (d) $E = \frac{n\hbar}{2}\sqrt{\frac{k}{m}}$

24. Which of the following statement(s) is (are) correct about the spectrum of hydrogen atom ? **[Adv. 2021]**

(a) The ratio of the longest wavelength to the shortest wavelength in Balmer series is 9/5

(b) There is an overlap between the wavelength ranges of Balmer and Paschen series.

(c) The wavelengths of Lyman series are given by $\left(1+\frac{1}{m^2}\right)\lambda_0$, where λ_0 is the shortest wavelength of Lyman series and m is an integer

(d) The wavelength ranges of Lyman and Balmer series do not overlap

25. A particle of mass m moves in circular orbits with potential energy $V(r) = Fr$, where F is a positive constant and r is its distance from the origin. Its energies are calculated using the Bohr model. If the radius of the particle's orbit is denoted by R and its speed and energy are denoted by v and E, respectively, then for the n^{th} orbit (here h is the Planck's constant) **[Adv. 2020]**

(a) $R \propto n^{1/3}$ and $v \propto n^{2/3}$ (b) $R \propto n^{2/3}$ and $v \propto n^{1/3}$

(c) $E = \frac{3}{2}\left(\frac{n^2h^2F^2}{4\pi^2 m}\right)^{1/3}$ (d) $E = 2\left(\frac{n^2h^2F^2}{4\pi^2 m}\right)^{1/3}$

26. A free hydrogen atom after absorbing a photon of wavelength λ_a gets excited from the state $n = 1$ to the state $n = 4$. Immediately after that the electron jumps to $n = m$ state by emitting a photon of wavelength λ_e. Let the change in momentum of atom due to the absorption and the emission are Δp_a and Δp_e, respectively. If $\lambda_a / \lambda_e = \frac{1}{5}$, which of the option(s) is/are correct?

[Use $hc = 1242$ eV nm; 1 nm = 10^{-9} m, h and c are Planck's constant and speed of light, respectively] **[Adv. 2019]**

(a) $\Delta p_a / \Delta p_e = \frac{1}{2}$

(b) The ratio of kinetic energy of the electron in the state $n = m$ to the state $n = 1$ is $\frac{1}{4}$

(c) $m = 2$

(d) $\lambda_e = 418$ nm

27. Highly excited states for hydrogen–like atoms (also called Rydberg states) with nuclear charge Ze are defined by their principal quantum number n, where n>>1. Which of the following statement(s) is(are) true? **[Adv. 2016]**

(a) Relative change in the radii of two consecutive orbitals does not depend on Z

(b) Relative change in the radii of two consecutive orbitals varies as 1/n

(c) Relative change in the energy of two consecutive orbitals varies as $1/n^3$

(d) Relative change in the angular momenta of two consecutive orbitals varies as 1/n

28. The radius of the orbit of an electron in a Hydrogen-like atom is 4.5 a_0, where a_0 is the Bohr radius. Its orbital angular momentum is $\frac{3h}{2\pi}$. It is given that h is Planck constant and R is Rydberg constant. The possible wavelength(s), when the atom de-excites, is (are) **[Adv. 2013]**

(a) $\frac{9}{32R}$ (b) $\frac{9}{16R}$ (c) $\frac{9}{5R}$ (d) $\frac{4}{3R}$

29. The electron in a hydrogen atom makes a transition $n_1 \to n_2$ where n_1 and n_2 are the principal quantum numbers of the two states. Assume the Bohr model to be valid. The time period of the electron in the initial state is eight times that in the final state. The possible values of n_1 and n_2 are **[1998S - 2 Marks]**

(a) $n_1 = 4, n_2 = 2$ (b) $n_1 = 8, n_2 = 2$

(c) $n_1 = 8, n_2 = 1$ (d) $n_1 = 6, n_2 = 3$

30. In Bohr's model of the hydrogen atom **[1984- 2 Marks]**

(a) the radius of the nth orbit is proportional to n^2

(b) the total energy of the electron in nth orbit is inversely proportional to n

(c) the angular momentum of the electron in an orbit is an integral multiple of $\frac{h}{2\pi}$

(d) the magnitude of potential energy of the electron in any orbit is greater than its K.E.

7 Match the Following

31. Some laws / processes are given in **Column I**. Match these with the physical phenomena given in **Column II** and indicate your answer by darkening appropriate bubbles in the 4 × 4 matrix given in the ORS. **[2007]**

Column I	Column II
(A) Transition between two atomic energy levels	(p) Characteristic X-rays
(B) Electron emission from a material	(q) Photoelectric effect
(C) Mosley's law	(r) Hydrogen spectrum
(D) Change of photon energy into kinetic energy of electrons	(s) β-decay

8 Comprehension/Passage Based Questions

Passage-1

The key feature of Bohr's theory of spectrum of hydrogen atom is the quantization of angular momentum when an electron is revolving around a proton. We will extend this to a general rotational motion to find quantized rotational energy of a diatomic molecule assuming it to be rigid. The rule to be applied is Bohr's quantization condition. **[2010]**

32. A diatomic molecule has moment of inertia I. By Bohr's quantization condition its rotational energy in the n^{th} level (n = 0 is not allowed) is

(a) $\frac{1}{n^2}\left(\frac{h^2}{8\pi^2 I}\right)$ (b) $\frac{1}{n}\left(\frac{h^2}{8\pi^2 I}\right)$

(c) $n\left(\frac{h^2}{8\pi^2 I}\right)$ (d) $n^2\left(\frac{h^2}{8\pi^2 I}\right)$

33. It is found that the excitation frequency from ground to the first excited state of rotation for the CO molecule is close to $\frac{4}{\pi} \times 10^{11}$ Hz. Then the moment of inertia of CO molecule about its center of mass is close to

(Take $h = 2\pi \times 10^{-34}$ J s)

(a) 2.76×10^{-46} kg m^2 (b) 1.87×10^{-46} kg m^2

(c) 4.67×10^{-47} kg m^2 (d) 1.17×10^{-47} kg m^2

34. In a CO molecule, the distance between C (mass = 12 a.m.u.) and O (mass = 16 a.m.u.), where 1 a.m.u. $=\frac{5}{3}\times10^{-27}$ kg, is close to

(a) 2.4×10^{-10} m (b) 1.9×10^{-10} m

(c) 1.3×10^{-10} m (d) 4.4×10^{-11} m

Passage-2

When a particle is restricted to move along x - axis between $x = 0$ and $x = a$, where a is of nanometer dimension, its energy can take only certain specific values. The allowed energies of the particle moving in such a restricted region, correspond to the formation of standing waves with nodes at its ends $x = 0$ and $x = a$. The wavelength of this standing wave is related to the linear momentum p of the particle according to the de Broglie relation. The energy of the particle of mass m is related to its linear momentum as $E=\frac{p^2}{2m}$. Thus, the energy of the particle can be denoted by a quantum number 'n' taking values 1, 2, 3, ... (n = 1, called the ground state) corresponding to the number of loops in the standing wave.

Use the model described above to answer the following three questions for a particle moving in the line $x = 0$ to $x = a$. Take $h = 6.6\times10^{-34}\ Js$ and $e = 1.6\times10^{-19}\ C$.

35. The allowed energy for the particle for a particular value of n is proportional to **[2009]**

(a) a^{-2} (b) $a^{-3/2}$ (c) a^{-1} (d) a^2

36. If the mass of the particle is $m = 1.0\times10^{-30}$ kg and $a = 6.6$ nm, the energy of the particle in its ground state is closest to **[2009]**

(a) 0.8 meV (b) 8 meV

(c) 80 meV (d) 800 meV

37. The speed of the particle, that can take discrete values, is proportional to

(a) $n^{-3/2}$ (b) n^{-1} (c) $n^{1/2}$ (d) n

Passage-3

In a mixture of H-He$^+$ gas (He+ is singly ionized He atom), H atoms and He+ ions are excited to their respective first excited states. Subsequently, H atoms transfer their total excitation energy to He+ ions (by collisions). Assume that the Bohr model of atom is exactly valid. **[2008]**

38. The quantum number n of the state finally populated in He$^+$ ions is –

(a) 2 (b) 3 (c) 4 (d) 5

39. The wavelength of light emitted in the visible region by He+ ions after collisions with H atoms is –

(a) 6.5×10^{-7} m (b) 5.6×10^{-7} m

(c) 4.8×10^{-7} m (d) 4.0×10^{-7} m

40. The ratio of the kinetic energy of the n = 2 electron for the H atom to that of He$^+$ ion is –

(a) 1/4 (b) 1/2 (c) 1 (d) 2

10 Subjective Problems

41. In hydrogen-like atom ($z = 11$), nth line of Lyman series has wavelength λ. The de-Broglie's wavelength of electron in the level from which it originated is also λ. Find the value of n? **[2006 - 6M]**

42. The photons from the Balmer series in Hydrogen spectrum having wavelength between 450 nm to 700 nm are incident on a metal surface of work function 2 eV. Find the maximum kinetic energy of ejected electron. (Given hc = 1242 eV nm) **[2004 - 4 Marks]**

43. A hydrogen-like atom (described by the Bohr model) is observed to emit six wavelengths, originating from all possible transitions between a group of levels. These levels have energies between -0.85 eV and -0.544 eV (**including** both these values). **[2002 - 5 Marks]**

(a) Find the atomic number of the atom.

(b) Calculate the smallest wavelength emitted in these transitions.

(Take hc = 1240 eV-nm, ground state energy of hydrogen atom = –13.6 eV)

44. A hydrogen-like atom of atomic number Z is in an excited state of quantum number $2n$. It can emit a maximum energy photon of 204 eV. If it makes a transition to quantum state n, a photon of energy 40.8 eV is emitted. Find n, Z and the ground state energy (in eV) for this atom. Also calculate the minimum energy (in eV) that can be emitted by this atom during de-excitation. Ground state energy of hydrogen atom is –13.6 eV. **[2000 - 6 Marks]**

45. An electron, in a hydrogen-like atom, is in an excited state. It has a total energy of –3.4 eV. Calculate (i) the kinetic energy and (ii) the de Broglie wavelength of the electron. **[1996 - 3 Marks]**

46. A hydrogen like atom (atomic number Z) is in a higher excited state of quantum number n. The excited atom can make a transition to the first excited state by successively emitting two photons of energy 10.2 and 17.0 eV respectively. Alternately, the atom from the same excited state can make a transition to the second excited state by successively emitting two photons of energies 4.25 eV and 5.95 eV respectively. **[1994 - 6 Marks]**

Determine the values of n and Z. (Ionization energy of H-atom = 13.6 eV)

47. A particle of charge equal to that of an electron, – e, and mass 208 times the mass of the electron (called a mu-meson) moves in a circular orbit around a nucleus of charge + 3e. (Take the mass of the nucleus to be infinite). Assuming that the Bohr model of the atom is applicable to this system. **[1988 - 6 Marks]**

(i) Derive an expression for the radius of the nth Bohr orbit.

(ii) Find the value of n for which the radius of the orbit is approximately the same as that of the first Bohr orbit for the hydrogen atom.

(iii) Find the wavelength of the radiation emitted when the mu-meson jumps from the third orbit of the first orbit.

48. A double ionised Lithium atom is hydrogen-like with atomic number 3. **[1985 - 6 Marks]**
(i) Find the wavelength of the radiation required to excite the electron in Li^{++} from the first to the third Bohr orbit. (Ionisation energy of the hydrogen atom equals 13.6 eV.)
(ii) How many spectral lines are observed in the emission spectrum of the above excited system?

49. The ionization energy of a hydrogen like Bohr atom is 4 Rydbergs. (i) What is the wavelength of the radiation emitted when the electron jumps from the first excited state to the ground state ? (ii) What is the radius of the first orbit for this atom? **[1984- 4 Marks]**

50. Hydrogen atom in its ground state is excited by means of monochromatic radiation of wavelength 975Å. How many different lines are possible in the resulting spectrum ? Calculate the longest wavelength amongst them. You may assume the ionization energy for hydrogen atom as 13.6 eV.

Topic-3: Miscellaneous (Mixed Concepts) Problems

10 Subjective Problems

1. Light from a discharge tube containing hydrogen atoms falls on the surface of a piece of sodium. The kinetic energy of the fastest photoelectrons emitted from sodium is 0.73 eV. The work function for sodium is 1.82 eV. Find **[1992 - 10 Marks]**
(a) the energy of the photons causing the photoelectric emission,
(b) the quantum numbers of the two levels involved in the emission of these photons,
(c) the change in the angular momentum of the electron in the hydrogen atom in the above transition, and
(d) the recoil speed of the emitting atom assuming it to be at rest before the transition.
(Ionization potential of hydrogen is 13.6 eV)

2. Electrons in hydrogen like atom ($Z = 3$) make transitions from the fifth to the fourth orbit and from the fourth to the third orbit. The resulting radiations are incident normally on a metal plate and eject photoelectrons. The stopping potential for the photoelectrons ejected by the shorter wavelength is 3.95 volts. Calculate the work function of the metal and the stopping potential for the photoelectrons ejected by the longer wavelength.
(Rydberg constant = $1.094 \times 10^7 \text{ m}^{-1}$) **[1990 - 7 Marks]**

3. A gas of identical hydrogen-like atoms has some atoms in the lowest (ground) energy level A and some atoms in a particular upper (excited) energy level B and there are no atoms in any other energy level. The atoms of the gas make transition to a higher energy level by absorbing monochromatic light of photon energy 2.7 eV. Subsequently, the atoms emit radiation of only six different photon energies. Some of the emitted photons have energy 2.7 eV, some have energy more and some have less than 2.7 eV. **[1989 - 8 Marks]**
(i) Find the principal quantum number of the initially excited level B.
(ii) Find the ionization energy for the gas atoms.
(iii) Find the maximum and the minimum energies of the emitted photons.

4. A single electron orbits around a stationary nucleus of charge $+Ze$. Where Z is a constant and e is the magnitude of the electronic charge. It requires 47.2 eV to excite the electron from the second Bohr orbit to the third Bohr orbit. Find **[1981- 10 Marks]**
(i) The value of Z.
(ii) The energy required to excite the electron from the third to the fourth Bohr orbit.
(iii) The wavelength of the electromagnetic radiation required to remove the electron from the first Bohr orbit to infinity.
(iv) The kinetic energy, potential energy and the angular momentum of the electron in the first Bohr orbit.
(v) The radius of the first Bohr orbit.
(The ionization energy of hydrogen atom = 13.6 eV, Bohr radius = 5.3×10^{-11} metre, velocity of light = 3×10^8 m/sec. Planck's constant = 6.6×10^{-34} joules - sec).

Answer Key

Topic-1 : Atomic Structure and Rutherford's Nuclear Model

1. (c)

Topic-2 : Bohr's Model and the Spectra of the Hydrogen Atom

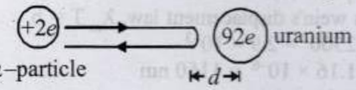
1. (a)	**2.** (d)	**3.** (a)	**4.** (b)	**5.** (a)	**6.** (d)	**7.** (a)	**8.** (b)	**9.** (d)	**10.** (a)
11. (c)	**12.** (b)	**13.** (d)	**14.** (d)	**15.** (3)	**16.** (3)	**17.** (5)	**18.** (6)	**19.** (2)	**20.** (2)
23. (a, b, c)	**24.** (a, d)	**25.** (b, c)	**26.** (b, c)	**27.** (a, b, d)	**28.** (a, c)	**29.** (a, d)	**30.** (a, c, d)		
31. A $\to$ p, r; B $\to$ q, s; C $\to$ p; D $\to$ q		**32.** (d)	**33.** (b)	**34.** (c)	**35.** (a)	**36.** (b)	**37.** (d)	**38.** (c)	
39. (c)	**40.** (a)								

Hints & Solutions

Topic-1: Atomic Structure and Rutherford's Nuclear Model

1. (c) Let 'r' be the distance of closest approach. One point charge is $\left({}^{235}_{92}U\right)$ uranium nucleus $\therefore\ q_1 = 92e$

Another point charge is α particle $\therefore\ q_2 = +2e$



Here loss in K.E. = gain in P.E. Energy conservation.

$$\therefore \frac{1}{2}mv^2 = k\frac{q_1q_2}{r} \Rightarrow r = k\frac{2q_1q_2}{\frac{1}{2}mv^2}$$

$$\therefore \quad r = \frac{9\times10^9\times2\times1.6\times10^{-19}\times92\times1.6\times10^{-19}}{5\times1.6\times10^{-13}}$$

$= 529.92 \times 10^{-16}$ m

Hence the distance of closest approach is of the order of 10^{-12} cm.

Topic-2: Bohr's Model and the Spectra of the Hydrogen Atom

1. (a) The ratio of the wavelengths of the kα - line and the cut-off i.e.

$$\frac{\lambda_{k\alpha}}{\lambda_0}(=r) \propto \frac{1}{(z-1)^2}$$

$$\therefore \frac{r_2}{r_1} = \frac{(z_1-1)^2}{(z_2-1)^2} = \frac{(46-1)^2}{(41-1)^2} = \frac{(45)^2}{(40)^2}$$

or, $r_2 = 1.265 \times r_1 = 1.265 \times 2 = 2.53$

2. (d) $\Delta E = hv$

$$v = \frac{\Delta E}{h} = k\left[\frac{1}{(n-1)^2} - \frac{1}{n^2}\right] = \frac{k(2n-1)}{n^2(n-1)^2} \approx \frac{2k}{n^3} \text{ or } v \propto \frac{1}{n^3}$$

3. (a) We know for hydrogen or hydrogen like atom,

$$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

For first spectral line in the Balmer series of hyrogen atom $n_1 = 2$ and $n_2 = 3$. Here $z = 1$

$$\therefore \frac{1}{6561} = R(1)^2\left(\frac{1}{4} - \frac{1}{9}\right) = \frac{5R}{36} \quad \text{...(i)}$$

For the second spectral line in the Balmer series of singly ionised helium ion $n_2 = 4$ and $n_1 = 2$; $Z = 2$

$$\therefore \frac{1}{\lambda} = R(2)^2\left[\frac{1}{4} - \frac{1}{16}\right] = \frac{3R}{4} \quad \text{...(ii)}$$

Dividing eq. (i) by (ii)

$$\frac{\lambda}{6561} = \frac{5R}{36} \times \frac{4}{3R} = \frac{5}{27} \qquad \therefore \quad \lambda = 1215\ \text{Å}$$

4. (b) The smallest frequency and longest wavelength in ultraviolet region will be for transition of electron from $n = 2$ to $n = 1$ i.e., Lyman series.

$$\therefore \quad \frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right)$$

$$\Rightarrow \frac{1}{122\times10^{-9}m} = R\left[\frac{1}{1^2} - \frac{1}{2^2}\right] = R\left[1 - \frac{1}{4}\right] = \frac{3R}{4}$$

$$\therefore R = \frac{4}{3\times122\times10^{-9}}m^{-1}$$

The highest frequency and smallest wavelength for infrared region will be for transition of electron $n = \infty$, $n = 3$ corresponds to paschem series.

$$\therefore \quad \frac{1}{\lambda} = R\left(\frac{1}{n_1^2} - \frac{1}{n_2^2}\right) \Rightarrow \frac{1}{\lambda} = \frac{4}{3\times122\times10^{-9}}\left(\frac{1}{3^2} - \frac{1}{\infty}\right)$$

$$\therefore \lambda = \frac{3\times122\times9\times10^{-9}}{4} = 823.5\text{nm}$$

5. (a) Initially a photon of energy 10.2eV collides inelastically with a hydrogen atom in ground state. For hydrogen atom,

$E_1 = -13.6$ eV; $E_2 = -\frac{13.6}{4}$ eV $= -3.4$eV

$\therefore\ E_2 - E_1 = 10.2$ eV

The electron of hydrogen atom will jump to second orbit after absorbing the photon of energy 10.2 eV. Another photon of energy 15 eV strikes the hydrogen atom inelastically. This energy is sufficient to knock out the electron from the atom as ionisation energy is 13.6 eV. The remaining energy $15 - 13.6 = 1.4$ eV is left which is released by the second photon.

6. (d) For an atom following Bohr's model, the radius of the fifth orbit.

$$r_m = \frac{r_0 m^2}{Z} \text{ where } r_0 = \text{Bohr's radius}$$

For ${}_{100}Fm^{257}$, $m = 5$ (Fifth orbit in which the outermost electron is present) and $z = 100$

$$\therefore r_m = \frac{r_0 \times 5^2}{100} = nr_0 \text{ (given)} \quad \therefore\ n = \frac{1}{4}$$

7. (a) Given potential energy (U) between electron and proton

$$= eV_0 \ln\frac{r}{r_0} \qquad [\because |U| = eV]$$

$$\therefore \quad |F| = \left|\frac{-du}{dr}\right| = \frac{d}{dr}\left[eV_0 \log_e\frac{r}{r_0}\right] = \frac{eV_0}{r_0} \times \frac{1}{r}$$

This force will provide the necessary centripetal force

$$\therefore \quad \frac{mv^2}{r} = \frac{eV_0}{rr_0} \Rightarrow mv^2 = \frac{eV_0}{r_0} \quad \text{...(i)}$$

As per Bohr's postulate, $mvr = \frac{nh}{2\pi}$ ....(ii)

From eq. (i) and (ii),

$$\frac{m^2v^2r^2}{mv^2} = \frac{n^2h^2r_0}{4\pi^2 \times V_0 e} \Rightarrow r^2 = \frac{n^2h^2r_0}{4\pi V_0 me} \Rightarrow r \propto n$$

8. (b) $l = \frac{nh}{2\pi}, |E| \propto Z^2/n^2; n = 3$

$\Rightarrow l_H = l_{Li}$ and $|E_H| < |E_{Li}|$

9. (d) For state transition, 2 to 1, 3 to 2 and 4 to 2 we get energy that $n = 4$ to $n = 3$,
Infrared radiation has less energy and greater than ultraviolet radiation.
Infrared radiation will be obtained in the transition 5 to 4.

10. (a) According to question, in a hydrogen atom makes atransition from an excited state to the ground state i.e., electron comes nearer to the nucleus so r decreases.

$\therefore$ Potential energy P.E. $= \frac{-Kze^2}{r}$ decreases

Kinetic energy K.E. will increase $\because$ K.E $= \frac{1}{2}\frac{kZe^2}{r}$

$\therefore$ Total energy decreases

$\because$ T.E.= P.E.+K.E. $= -\frac{1}{2}\frac{kZe^2}{r}$

11. (c) For ordinary hydrogen atom, longest wavelength

$$\frac{1}{\lambda} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right] = \frac{5R}{36} \text{ or } \lambda = \frac{36}{5R}$$

With hypothetical particle, required wavelength

$$\lambda' = \frac{1}{2} \times \frac{36}{5R} = \frac{18}{5R} \quad \because \lambda \propto \frac{1}{m}$$

12. (b) According to Doppler's effect of light, the wavelength shift

$$\Delta\lambda = \frac{v}{c} \times \lambda \Rightarrow v = \frac{\Delta\lambda \times c}{\lambda}$$

$$\therefore V = \frac{(706-656)}{656} \times 3 \times 10^8 \approx 2 \times 10^7 \text{ m/s}$$

13. (d) Ground state energy of doubly ionized lithium atom ($Z = 3, n = 1$)

$$E_1 = -13.6\frac{(Z^2)}{(n^2)}\text{eV} = (-13.6)\frac{(3)^2}{(1)^2} = -122.4 \text{ eV}$$

$\therefore$ Ionization energy or the minimum energy of an electron in ground state of doubly ionized lithium atom will be 122.4eV.

14. (d) As we know, $\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_2^2} - \frac{1}{n_1^2}\right] \Rightarrow \frac{1}{\lambda} \propto Z^2$

λ is shortest when Z is highest. Z is highest for doubly ionised lithium.

15. (3) $K_{max} = E - W \Rightarrow E_{4\to3} = K_{max} + W = 1.95 + \frac{hc}{\lambda}$

$$= 1.95 + \frac{1240}{310} = 5.95\text{eV}$$

$$13.6Z^2\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = 5.95$$

$$13.6Z^2\left(\frac{7}{9\times16}\right) = 5.95 \Rightarrow Z^2 = \frac{5.95 \times 9 \times 16}{13.6 \times 7} = 9$$

$\therefore Z = 3$

16. (3) According to question, the photon emitted $n = 2 \to n = 1$ transition has energy 74.8 eV higher than the photon emitted $n = 3 \to n = 2$

$\therefore \quad \Delta E_{2-1} = 74.8 + \Delta E_{3-2}$

$$13.6z^2\left[1 - \frac{1}{4}\right] = 74.8 + 13.6z^2\left[\frac{1}{4} - \frac{1}{9}\right] \quad \therefore z = 3$$

17. (5) Here $\frac{V_i}{V_f} = \frac{\frac{-27.2}{n_f^2}}{\frac{-27.2}{n_i^2}} = \frac{n_f^2}{n_i^2} = 6.25 \quad \therefore \frac{n_f}{n_i} = 2.5 = \frac{5}{2}$

$\therefore$ Smallest possible $n_f = 5$

18. (6) Energy of incident light,

$$E = \frac{hc}{\lambda} = \frac{1.237 \times 10^{-6}}{970 \times 10^{-10}}eV = 12.75eV$$

$\therefore$ The energy of electron after absorbing this photon $= -13.6 + 12.75 = -0.85$eV

Let the electron tumps to nth state after excitation

$\therefore \frac{-13.6}{n^2} = -0.85 \therefore n = 4$

$\therefore$ Total number of spectral line $= \frac{n(n-1)}{2} = \frac{4(4-1)}{2} = 6$

19. (2) Angular momentum, $L = mvr = \frac{nh}{2\pi} = \frac{3h}{2\pi} \Rightarrow n = 3$

$\because \lambda = \frac{h}{p} = \frac{h}{mv} = \frac{hr}{mvr} \Rightarrow mvr = \frac{hr}{\lambda} \quad \therefore \frac{hr}{\lambda} = \frac{3h}{2\pi}$

$\therefore \quad \lambda = \frac{2\pi r}{3} = \frac{2}{3}\pi\left[a_0\frac{n^2}{z}\right] \quad \left[\because r = a_0\frac{n^2}{z}\right]$

$\therefore \quad \lambda = \frac{2}{3}\pi a_0\left[\frac{3\times3}{3}\right] = 2\pi a_0 \quad [\because n = 3; z = 3] \quad \therefore p = 2$

20. (2) Using energy conservation principle,

$$E_{photon} = E_{ionize} + E_k \Rightarrow \frac{hc}{\lambda} = \frac{13.6}{n^2} + E_k$$

$$\therefore \frac{1242}{90} = \frac{13.6}{n^2} + 10.2 \Rightarrow n^2 = 4 \quad \therefore n = 2$$

21. In the bohr model of hydrogen atom, kinetic energy,
K.E. $= \frac{kZe^2}{2r}$ and

Total energy, T.E. $= \frac{-kZe^2}{2r} \quad \therefore \frac{\text{K.E.}}{\text{T.E.}} = -1$

22. The fifth valence electron of phosphorous is in its third shell, i.e., $n = 3$. For phosphorous, $Z = 15$.
$\therefore$ Bohr's radius for nth orbit

$$r_n = \left(\frac{n^2}{Z}\varepsilon_r\right)r_0 = \frac{3^2}{15} \times 12 \times 0.529 \text{ Å} = 3.81\text{Å}$$

23. (a, b, c) Mass m is moving in a circular orbit under the influence of the central force $F(r) = -kr$ which provide the necessary centripetal force

$\therefore \quad F(r) = F(c)$ or, $kr = \frac{mv^2}{r}$

or, $kr^2 = mv^2$ ..... (i)

using quantisation rule $n\hbar = mvr$

or, $\frac{n\hbar}{r} = mv$ ....(ii)

Squaring both sides of eq. (ii) $\frac{n^2\hbar^2}{r^2} = m^2v^2$ .... (iii)

Dividing eq. (i) by (ii)

$$\frac{kr^2}{\frac{n^2\hbar^2}{r^2}} = \frac{mv^2}{m^2v^2}$$

$$\Rightarrow \quad \frac{k}{n^2\hbar^2}r^4 = \frac{1}{m}$$

$$\Rightarrow \quad r = \left(\frac{n^2\hbar^2}{km}\right)^{\frac{1}{4}} \Rightarrow r^2 = n\hbar\sqrt{\frac{1}{mk}}$$ so option (a) is correct

NOW using eq. (i) $k \cdot \frac{n\hbar}{\sqrt{mk}} = mv^2 \Rightarrow v^2 = n\hbar\sqrt{\frac{k}{m^3}}$

So, option (b) is correct

Again using eq. (i) $\frac{L}{mr^2} = \frac{mvr}{mr^2} = \frac{v}{r} = \sqrt{\frac{k}{m}}$

so option (c) is correct

Total energy, E = K + E or,

$$E = \frac{1}{2}mv^2 + \frac{1}{2}kr^2 = \frac{n\hbar}{2}\sqrt{\frac{k}{m}} + \frac{1}{2}k\frac{n\hbar}{\sqrt{mk}}$$

$$\therefore \quad E = n\hbar\sqrt{\frac{k}{m}}$$

so option (d) in incorrect.

24. **(a, d)** From formula,

(a) $\frac{1}{\lambda} = R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$

For Balmer series, $n_1 = 2$

$\therefore$ For longest wavelength, transition occurs from $n = 3$ to $n = 2$.

$$\therefore \quad \frac{1}{\lambda_{max}} = R\left[\frac{1}{2^2} - \frac{1}{3^2}\right]$$

& for shortest wavelength transition occurs from $n = \infty$ to $n = 2$

$$\therefore \quad \frac{1}{\lambda_{min}} = R\left[\frac{1}{2^2} - \frac{1}{\infty^2}\right] \quad \therefore \quad \frac{\lambda_{longest}}{\lambda_{shorts}} = \frac{9}{5}$$

(b) For Paschen series, $n_1 = 3$

$\lambda_{longest}$ of Balmer $= \frac{36}{5R}$ and

$\lambda_{shortest}$ of Paschen $= \frac{9}{R}$

Hence there is no overlap between the wavelength ranges of Balmer and Paschen series.

(c, d) For Lyman series, $n_1 = 1$

$$\frac{1}{\lambda} = R\left[\frac{1}{1} - \frac{1}{m^2}\right]$$

Also $\frac{1}{\lambda_0} = R \quad \therefore \quad \frac{1}{\lambda} = \frac{1}{\lambda_0}\left[1 - \frac{1}{m^2}\right] \Rightarrow \lambda = \frac{\lambda_0}{1 - \frac{1}{m^2}}$

$\lambda_{longest}$ of Lyman $= \frac{4}{3R}$ and $\lambda_{shortest}$ of Balmer $= \frac{4}{R}$

Hence that wavelenght ranges of Lyman and Balmer series do not overlap.

25. **(b, c)** Given: Potential energy of the particle of mass 'm' moving in circular orbit, V(r) = Fr

$$\Rightarrow F = \frac{V(r)}{r}$$

Also centripetal force

$$F = \frac{mv^2}{R} \qquad ...(1)$$

According to Bohr's second postulate,

$$mvR = \frac{nh}{2\pi} \Rightarrow V = \frac{nh}{2\pi mr}$$

Putting this value of v in eqn (1)

$$F = \frac{m}{R} \times \frac{n^2h^2}{2\pi^2} \times \frac{1}{m^2R^2} \Rightarrow R = \left(\frac{n^2h^2}{4\pi^2 mR}\right)^{1/3} \quad \therefore R \propto n^{2/3}$$

Now putting this value of R in $v = \frac{nh}{2\pi mR}$

$$v = \frac{nh}{2\pi m}\left(\frac{4\pi^2 mF}{n^2h^2}\right)^{1/3} \quad \therefore v \propto n^{1/3}$$

Hence, option (b) is correct.

Total energy $E = \frac{1}{2}mv^2 + v(r) = \frac{1}{2}mv^2 + FR$

$$\Rightarrow E = \frac{1}{2}m\left(\frac{n^{2/3}h^{2/3}F^{2/3}}{2^{2/3}\pi^{2/3}m^{4/3}}\right) + F \times \left(\frac{n^2h^2}{4\pi^2 mF}\right)^{1/3}$$

$$\Rightarrow E = \left(\frac{n^2h^2F^2}{4\pi^2 m}\right)^{1/3}\left[\frac{1}{2} + 1\right] \text{ or, } E = \frac{3}{2}\left(\frac{n^2h^2F^2}{4\pi^2 m}\right)^{1/3}$$

Hence, option (c) is correct.

26. **(b, c)** (a) Change in linear momentum due to absorption

$$\Delta P_a = \frac{h}{\lambda a} \quad \left(\because \lambda = \frac{h}{P}\right)$$

Change in linear momentum due to emission

$$\Delta P_e = \frac{h}{\lambda e} \quad \because \frac{\Delta P_a}{\Delta P_e} = \frac{\lambda_e}{\lambda_a} = 5\left(\because \frac{\lambda_a}{\lambda_e} = \frac{1}{5}\right) \text{ given}$$

So, option (a) is wrong.

(b) Kinetic energy $K_n \propto \frac{z^2}{n^2} \therefore \frac{K_2}{K_1} = \frac{1^2}{2^2} = \frac{1}{4}$

So, (b) is correct.

(c) For absorption of energy, $n = 1$ to $n = 4$

$$E_4 - E_1 = \frac{hc}{\lambda_a} = 13.6\left(\frac{1}{1} - \frac{1}{4^2}\right) \qquad ...(i)$$

Per emission of energy, $n = m$ to $n = 4$

$$E_4 - E_m = \frac{h_c}{\lambda e} = 13.6\left(\frac{1}{m^2} - \frac{1}{4^2}\right) \qquad ...(ii)$$

Dividing eq (ii) by (i)

$$\frac{\lambda a}{\lambda e} = \frac{\frac{1}{m^2} - \frac{1}{16}}{1 - \frac{1}{16}} = \frac{1}{5}$$

$$\Rightarrow \frac{1}{m^2} - \frac{1}{16} = \frac{1}{5} \times \frac{15}{16} \Rightarrow \frac{1}{m^2} = \frac{3}{16} + \frac{1}{16} = \frac{4}{16}$$

$\therefore m = 2$. So (c) is correct.

(d) Now from eq. (ii)

$$\frac{hc}{\lambda e} = 13.6\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = 13.6\left(\frac{1}{4} - \frac{1}{16}\right)$$

$$\therefore \lambda e = \frac{1242 \times 16}{13.6 \times 3} \approx 486.2 nm.$$ So (d) is wrong.

27. **(a, b, d)** We know radius, $r_n = r_0\frac{n^2}{z}$, Energy $E_n = -\frac{13.6Z^2}{n^2}$, angular momentum, $L_n = \frac{nh}{2\pi}$

Relative change in the radii of two consecutive orbitals

$$\frac{\Delta r}{r} = \frac{r_n - r_{n-1}}{r_n} = 1 - \frac{r_{n-1}}{r_n} = 1 - \frac{(n-1)^2}{n^2}$$

$$= \frac{2n-1}{n^2} \approx \frac{2}{n} \quad (\because n >> 1), \text{ which is independent of Z.}$$

Relative change in the energy of two consecutive orbitals

$$\frac{\Delta E}{E} = \frac{E_n - E_{n-1}}{E_n} = 1 - \frac{E_{n-1}}{E_n} = 1 - \frac{n^2}{(n-1)^2} = \frac{-2n+1}{(n-1)^2} \approx \frac{-2}{n}$$

$$\frac{\Delta L}{L} = \frac{L_n - L_{n-1}}{L_n} = 1 - \frac{L_{n-1}}{L_n} = 1 - \frac{(n-1)}{n} = \frac{1}{n}$$

28. **(a, c)** According to Bohr's quantisation, angular momentum

$$L = \frac{nh}{2\pi} = \frac{3h}{2\pi} \therefore n = 3.$$

Also $r_n = \frac{a_o n^2}{z} = 4.5 a_o$

$$\therefore \frac{n^2}{z} = 4.5 \Rightarrow \frac{9}{z} = 4.5 \Rightarrow z = 2$$

According to Rydberg formula

$$\frac{1}{\lambda} = R z^2 \left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = \frac{1}{\lambda} = 4R\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$

For $n_2 = 3, \rightarrow n_1 = 1$ $\lambda = \frac{9}{8 \times 4R} = \frac{9}{32R}$

For $n_2 = 3, \rightarrow n_1 = 2$ $\lambda = \frac{36}{5 \times 4R} = \frac{9}{5R}$

For $n_2 = 2, \rightarrow n_1 = 1$ $\lambda = \frac{4}{3 \times 4R} = \frac{1}{3R}$

29. **(a, d)** The time period of the electron in a Bohr orbit in nth state $T_n = \frac{2\pi rn}{V_n}$ or, $T_n \propto \frac{r_n}{V_n}$ ...(i)

Bohr radius of a hydrogen atom

$$r_n = n^2\left(\frac{h^2 \varepsilon_0}{\pi m e^2}\right) \text{ or, } r_n \propto n_2 \quad \text{...(ii)}$$

And, $V_n \propto \frac{1}{n}$ ...(iii)

From eq. (i), (ii) and (iii) $T_n \propto n^3$.

And according to question, $Tn_1 = 8T_2 \therefore n_1 = 2n_2$

If $n_2 = 2 \Rightarrow n_3 = 4$

$n_2 = 3 \Rightarrow n_3 = 6$

30. **(a, c, d)** Radius of nth orbit, $r_n \propto n^2$

$$E_n = \frac{-13.6Z^2}{n^2} \text{eV}$$

Angular momentum, $L_n = mvr = \frac{nh}{2\pi}$

$|\text{P.E.}| = 2 \times |\text{K.E.}|$

31. **A → p, r**

The lines in the hydrogen spectrum is obtained due to transition of electrons from one energy level to another. Characteristic X-ray are produced due to transition of electrons from one energy level to another.

B → q, s

In photoelectric effect electrons from the metal surface are emitted when light of appropriate frequency incident on it.

In β-decay, electrons are emitted from the nucleus of an atom.

C → p

According to Moseley law frequency of emitted X-ray is related with the atomic number (z) of the target material as $\sqrt{\nu} = a(Z - b)$

D → q

According to Einstein's photoelectric effect, energy of photons of incident ray gets converted into kinetic energy of emitted electrons K. $E_{max} = h\nu - \phi$

32. **(d)** Rotational kinetic energy $K{\cdot}E_{rot} = \frac{1}{2}I\omega^2 = \frac{1}{2}I\left[\frac{L}{I}\right]^2$

$[\because L = I\omega]$

And according to Bohr's quantisation principle $L = \frac{nh}{2\pi}$

$$\therefore K{\cdot}E_{rot} = \frac{1}{2}\frac{L^2}{I} = \frac{1}{2I} \times \frac{n^2h^2}{4\pi^2} = n^2\left[\frac{h^2}{8\pi^2 I}\right] \quad \text{...(i)}$$

33. **(b)** From ground ($n = 1$) to first excited state ($n = 2$)

Energy given = change in kinetic energy

$$h\nu = K_f - K_i = \frac{h^2}{8\pi^2 I}[2^2 - 1^2] \quad \text{[From eq. (i)]}$$

$$h\nu = \frac{3h^2}{8\pi^2 I} \Rightarrow I = \frac{3h}{8\pi^2\nu} = \frac{3 \times 2\pi \times 10^{-34}}{8\pi^2 \times \frac{4}{\pi} \times 10^{11}} = \frac{3}{16} \times 10^{-45}$$

$= 1.87 \times 10^{-46}$ kg m^2

34. **(c)** Moment of inertia of CO molecule,

$$I = \mu r^2 \Rightarrow r^2 = \frac{I}{\mu}$$

where, μ = reduced mass of the CO molecule, r = distance between C and O

Reduced mass of the CO molecule

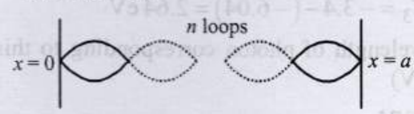
$$\mu = \frac{m_1 m_2}{m_1 + m_2} = \left[\frac{(12)(16)}{12+16}\right] \times \frac{5}{3} \times 10^{-27} \text{kg}$$

But $I = 1.87 \times 10^{-46}$ kg m^2 (from the above question)

$$\therefore r^2 = \left[\frac{1.87 \times 10^{-46}}{\frac{12 \times 16}{28} \times \frac{5}{3} \times 10^{-27}}\right] = \frac{1.87 \times 10^{-46} \times 28 \times 3}{12 \times 16 \times 5 \times 10^{-27}}$$

$\therefore r = 1.3 \times 10^{-10}$ m

35. **(a)**



Energy, $E = \frac{h^2}{2m\lambda^2}$ $\left(\because E = \frac{P^2}{2m} \text{ and } P = \frac{h}{\lambda}\right)$

The length in which the particle is restricted to move is

$$a = n\frac{\lambda}{2} \Rightarrow \lambda = \frac{2a}{n}$$

Putting this value of λ we get

$$E=\frac{h^2n^2}{2m\times 4a^2}=\frac{n^2h^2}{8ma^2} \qquad \therefore E\propto a^{-2}$$

36. **(b)** For ground state $n = 1$,
Given $m = 1.0 \times 10^{-30}$ kg, $a = 6.6 \times 10^{-9}$ m

$$\therefore E=\frac{n^2h^2}{8ma^2}=\frac{1^2\times(6.6\times10^{-34})^2}{8\times1\times10^{-30}\times(6.6\times10^{-9})^2}\text{J}=8\text{ meV}$$

37. **(d)** $\lambda=\frac{h}{p} \Rightarrow \lambda=\frac{h}{mv}$ But $\frac{n\lambda}{2}=a \Rightarrow \lambda=\frac{2a}{n}$

$$\therefore \frac{h}{mv}=\frac{2a}{n} \quad \therefore mv=\frac{nh}{2a} \Rightarrow v=\frac{nh}{2am} \quad \therefore v\propto n$$

38. **(c)** For hydrogen or hydrogen like atoms

$$E_n=\frac{-13.6\,Z^2}{n^2}\text{eV/atom}$$

For hydrogen atom $E_1 = -13.6$eV (for $n = 1$)
(Z = 1) $E_2 = -3.4$eV (for $n = 2$)

$\therefore \Delta E = E_2 - E_1 = -3.4 - (-13.6) = 10.2$ eV

i.e., When hydrogen comes to ground state from its first excited state it will release 10.2 eV of energy.

For He^+ ion $E_1 = -13.6 \times 4$ eV $= -54.4$ eV (for $n = 1$)
(Z = 2) $E_2 = -13.6$ eV (for $n = 2$)
$E_3 = -6.04$ eV (for $n = 3$)
$E_4 = -3.4$eV (for $n = 4$)

Here He^+ ion is in the first excited state i.e., possessing energy – 13.6 eV. After receiving energy of + 10.2 eV from excited hydrogen atom on collision, the energy of electron will be (–13.6 + 10.2) eV = –3.4 eV. Hence the quantum number of the state finally populated in He^+ ions, $n = 4$.

39. **(c)** Wavelength of visible light lies in the range, $\lambda_1 = 4000$Å to $\lambda_2 = 7000$Å.

Therefore $E_1=\frac{12375}{\lambda_1}=\frac{12375}{4000}=3.09$ eV

$$E_2=\frac{12375}{\lambda_2}=\frac{12375}{7000}=1.77\text{ eV}$$

For He^+ atom in transition from $n = 4$ to $n = 3$, energy of photon released will lie between E_1 and E_2.

$E_4 - E_3 = -3.4 - (-6.04) = 2.64$ eV

$\therefore$ Wavelength of photon corresponding to this energy, (2.64 eV)

$$\lambda=\frac{12375}{2.64}\text{Å}=4687.5\text{ Å}=4.68\times10^{-7}\text{ m}$$

40. **(a)** Kinetic energy for hydrogen or hydrogen like atom K

$$K=\frac{-13.6Z^2}{n^2} \Rightarrow K\propto Z^2$$

$$\frac{K_H}{K_{He^+}}=\left(\frac{Z_H}{Z_{He^+}}\right)^2=\left(\frac{1}{2}\right)^2=\frac{1}{4}$$

41. For an electron to revolve in $(n + 1)$th orbit.
$2\pi r = (n + 1)\lambda$

$$\Rightarrow \lambda=\frac{2\pi}{(n+1)}\times r=\frac{2\pi}{(n+1)}\left[0.529\times10^{-10}\right]\frac{(n+1)^2}{Z}$$

$$\Rightarrow \frac{1}{\lambda}=\frac{Z}{2\pi\left[0.529\times10^{-10}\right](n+1)} \quad ...(i)$$

And when electron jumps from $(n + 1)$th to 1st orbit.

$$\frac{1}{\lambda}=RZ^2\left[\frac{1}{1^2}-\frac{1}{(n+1)^2}\right]=1.09\times10^7Z^2\left[1-\frac{1}{(n+1)^2}\right] \quad ...(ii)$$

From eq. (i) and (ii)

$$\frac{Z}{2\pi(0.529\times10^{-10})(n+1)}=1.09\times10^7Z^2\left[1-\frac{1}{(n+1)^2}\right]$$

Putting the value of $z = 11$ and solving, we get $n = 24$

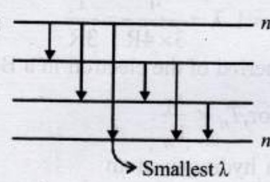
42. For different lines of Balmer series

$$\frac{hc}{\lambda}=13.6\left[\frac{1}{2^2}-\frac{1}{n^2}\right]\text{ eV, where } n=3, 4, 5$$

Solving, we get $\lambda = 657$nm, 487 nm between 450 nm and 700 nm. Wavelength 487 nm < 657 nm, electron of max. K.E. will be emitted for photon corresponding to wavelength 487 nm with

$$(\text{K.E.})=\frac{hc}{\lambda}-W=\left(\frac{1242}{487}-2\right)=0.55\text{ eV}$$

43. **(a)** Let



We have $\frac{-z^2(13.6\text{eV})}{n^2}=-0.85$eV ...(i)

and $\frac{-z^2(13.6\text{eV})}{(n+3)^2}=-0.544$ eV ...(ii)

From eq. (i) and (ii) n = 12 and z = 3

(b) Smallest wavelength λ

$$\frac{hc}{\lambda}=(0.85-0.544)\text{ eV}$$

Putting the value of h, and c and solving, we get $\lambda = 4052$ nm.

44. Energy for an orbit of hydrogen or hydrogen like atom

$$E_n=-\frac{13.6Z^2}{n^2}$$

For transition from $n = 2n$ orbit to $n = 1$ orbit

Maximum energy, $E_{max}=13.6Z^2\left(\frac{1}{1}-\frac{1}{(2n)^2}\right)=204$

Also for transition $n = 2n$ to $n = n$.

$$40.8 = 13.6Z^2\left(\frac{1}{n^2}-\frac{1}{4n^2}\right) \Rightarrow 40.8 = 13.6Z^2\left(\frac{3}{4n^2}\right)$$

$$\Rightarrow 40.8 = 40.8\,\frac{Z^2}{4n^2} \Rightarrow 4n^2 = Z^2 \text{ or } 2n = Z \text{ ....(ii)}$$

From eq. (i) and (ii)

$$204 = 13.6Z^2\left(1-\frac{1}{Z^2}\right) = 13.6Z^2 - 13.6$$

$$13.6Z^2 = 204 + 13.6 = 217.6 \Rightarrow Z^2 = \frac{217.6}{13.6} = 16, \therefore Z = 4$$

$$n = \frac{Z}{2} = \frac{4}{2} = 2$$

For minimum energy = transition from 4 to 3.

$$E_{\min} = 13.6\times 4^2\left(\frac{1}{3^2}-\frac{1}{4^2}\right) = 13.6\times 4^2\left(\frac{7}{9\times 16}\right) = 10.5 \text{ eV.}$$

45. (i) Kinetic energy = total energy
$\therefore$ Kinetic energy, K.E. = 3.4 eV
(ii) The de Broglie wavelength of electron

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{6.64\times 10^{-34}}{\sqrt{2\times 9.1\times 10^{-31}\times 3.4\times 1.6\times 10^{-19}}} \text{ eV}$$

$= 0.66\times 10^{-9}$ m = 6.6Å

46. For hydrogen or hydrogen like atoms

$$E_n = -\frac{13.6}{n^2}Z^2 \text{ eV/atom}$$

From the condition given,

$E_n - E_2 = 10.2 + 17 = 27.2$ eV ..(i)
$E_n - E_3 = 4.24 + 5.95 = 10.2$ eV
$\therefore E_3 - E_2 = 17$

$$\text{But } E_3 - E_2 = -\frac{13.6}{9}Z^2 - \left(-\frac{13.6}{4}Z^2\right)$$

$$= -13.6Z^2\left[\frac{1}{9}-\frac{1}{4}\right] = -13.6Z^2\left[\frac{4-9}{36}\right] = \frac{13.6\times 5}{36}Z^2$$

$$\therefore \frac{13.6\times 5}{36}Z^2 = 17 \Rightarrow Z = 3$$

$$E_n - E_2 = -\frac{13.6}{n_2}\times 3^2 - \left[-\frac{13.6}{2^2}\times 3^2\right]$$

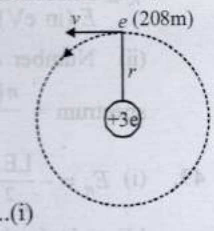
$$= -13.6\left[\frac{9}{n^2}-\frac{9}{4}\right] = -13.6\times 9\left[\frac{4-n^2}{4n^2}\right] \text{ ...(ii)}$$

From eq. (i) and (ii),

$$-13.6\times 9\left[\frac{4-n^2}{4n^2}\right] = 27.2 \Rightarrow -122.4(4-n^2) = 108.8n^2$$

$$\Rightarrow n^2 = \frac{489.6}{13.6} = 36 \therefore n = 6$$

47. (i) Let mass of electron = m
$\therefore$ Mass of mu-meson = $(208)m$
Here Bohr model of the atom is applicable to this system



$$\therefore \text{Angular momentum, } mur = \frac{nh}{2\pi}$$

$$\therefore \quad (208m)\,vr = \frac{nh}{2\pi}$$

$$\therefore \quad v = \frac{nh}{2\pi\times 208mr} = \frac{nh}{416\pi mr} \text{ ...(i)}$$

Since mu-meson is revolving in a circular path, around the nucleus which is of infinite mass and at rest therefore, it needs centripetal force which is provided by the electrostatic force between the nucleus and mu-meson.

$$\therefore \quad \frac{(208m)v^2}{r} = \frac{1}{4\pi\varepsilon_0}\times\frac{3e\times e}{r^2}$$

$$\therefore \quad r = \frac{3e^2}{4\pi\varepsilon_0\times 208mv^2}$$

Putting the value of v from eq. (i), we get

$$r = \frac{3e^2\times 416\pi mr\times 416\pi mr}{4\pi\varepsilon_0\times 208mn^2h^2}$$

Therefore, the radius of the n[th] Bohr orbit.

$$r = \frac{n^2h^2\varepsilon_0}{624\pi me^2} \text{ ...(ii)}$$

(ii) To find the value of n for which the radius of the orbit is approximately the same as that of the first Bohr orbit for hydrogen atom,
The radius of the first Bohr orbit of the hydrogen atom

$$= \frac{\varepsilon_0 h^2}{\pi me^2} \quad \therefore \frac{n^2h^2\varepsilon_0}{624\pi me^2} = \frac{\varepsilon_0 h^2}{\pi me^2} \quad \therefore n = \sqrt{624} \approx 25$$

(iii) The wavelength of the radiation emitted when mu-meson jumps from the third or bit ($n = 3$) to first orbit ($n = 1$).

$$\frac{1}{\lambda} = 208R\times Z^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$$

$$\Rightarrow \frac{1}{\lambda} = 208\times 1.097\times 10^7\times 3^2\left[\frac{1}{1^2}-\frac{1}{3^2}\right]$$

$\Rightarrow \lambda = 5.478\times 10^{-11}$m

48. (i) Wavelength of the radiation required to excite the electron in Li^{++} from $n_1 = 1$ to $n_2 = 3$

$$E_n = -\frac{13.6}{n^2}Z^2 \text{ eV/atom}$$

$$\text{For } Li^{2+}, Z = 3 \quad \therefore E_n = \frac{-13.6\times 9}{n^2} \text{ eV/atom}$$

$$\therefore E_1 = -\frac{13.6\times 9}{1} \text{ and } E_3 = -\frac{13.6\times 9}{9} = -13.6$$

$\Delta E = E_3 - E_1 = -13.6 - (-13.6\times 9)$

$= 13.6 \times 8 = 108.8$ eV/atom

$\lambda = \frac{12400}{E \text{ (in eV)}} Å = \frac{12400}{108.8} = 114 Å$

(ii) Number of spectral lines observed in the emission spectrum $= \frac{n(n-1)}{2} = \frac{3(3-1)}{2} = 3$

49. (i) $E_n = -\frac{\text{I.E.}}{n^2}$ for Bohr's hydrogen atom.

I.E. = Ionisation energy

Here, I.E. = $4R$ ($R = 1$ Rydberg $= Rhc = 2.2 \times 10^{-18}$ J)

$\therefore E_n = \frac{-4R}{n^2}$

$\therefore E_2 - E_1 = \frac{-4R}{2^2} - \left(-\frac{4R}{1^2}\right) = 3R$ ...(i)

$E_2 - E_1 = h\nu = \frac{hc}{\lambda}$ ...(ii)

From eq. (i) and (ii)

$\frac{hc}{\lambda} = 3R \therefore \lambda = \frac{hc}{3R}$

or, $\lambda = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{2.2 \times 10^{-18} \times 3} = 300 Å$

(ii) **The radius of the first orbit**

$|E_n| = +0.22 \times 10^{-17} Z^2 = 4R = 4 \times 2.2 \times 10^{-18} \therefore Z = 2$

$\therefore r_1 = \frac{r_0}{Z} = \frac{5 \times 10^{-11}}{Z} = \frac{5 \times 10^{-11}}{2} = 2.5 \times 10^{-11}$m

50. Energy $E = \frac{h_c}{\lambda} \Rightarrow E = \frac{12400}{\lambda(\text{in} Å)}$ eV $= \frac{12400}{975} = 12.75$ eV ...(i)

Let the electron excites from $n_1 = 1$ to n_2 state

$13.6\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right] = 12.75 \Rightarrow \left[\frac{1}{1} - \frac{1}{n_2^2}\right] = \frac{12.75}{13.6} \Rightarrow n_2 \approx 4$

Total number of lines in emission spectrum $= \frac{n(n-1)}{2}$

$= \frac{4(4-1)}{2} = 6$

For longest wavelength, the frequency should be smallest. This corresponds to the transition from $n = 4$ to $n = 3$,

$E_4 = -\frac{13.6}{4^2}; E_3 = -\frac{13.6}{3^2}$

$\therefore E_4 - E_3 = \frac{-13.6}{4^2} - \left(\frac{-13.6}{3^2}\right) = 13.6\left[\frac{1}{9} - \frac{1}{16}\right] = 0.66$ eV

Now, $\because E = \frac{12400}{\lambda(\text{in} Å)}$ eV $\therefore \lambda = \frac{12400}{E} = \frac{12400}{0.66} Å$

$= 1.875 \times 10^{-6}$ m

Topic-3: Miscellaneous (Mixed Concepts) Problems

1. (a) From Einstein's photoelectric equation, energy of photon causing photoelectric emission (E) = Work function of sodium metal + KE of the fastest photoelectron K_{max}.

$= 1.82 + 0.73 = 2.55$ eV

(b) For hydrogen of hydrogen like atom

$E_n = \frac{-13.6}{n^2} \frac{\text{eV}}{\text{atom}}$

For $n = 1$, $E_1 = -13.6$ ev

For $n = 2$, $E_2 = -3.4$ ev

For $n = 3$, $E_3 = -1.5$ ev

For $n = 4$, $E_4 = -0.85$ ev

Here, $E_4 - E_2 = 2.55$ ev

Hence the quantum numbers of the two levels involved in the emission of these photons are from $n = 4$ to $n = 2$

(c) Change in angular momentum in transition from $n = 4$ to $n = 2$

$\Delta L = \frac{n_1 h}{2\pi} - \frac{n_2 h}{2\pi} = \frac{h}{2\pi}(2-4) = \frac{h}{2\pi} \times (-2) = -\frac{h}{\pi}$

(d) The momentum of emitted photon can be found by de Broglie relationship

$\lambda = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda} = \frac{h\nu}{c} = \frac{E}{c} \therefore p = \frac{2.55 \times 1.6 \times 10^{-19}}{3 \times 10^8}$

According to conservation of linear momentum, momentum of emitted photon = momentum of hydrogen atom

$\therefore m \times v = \frac{2.55 \times 1.6 \times 10^{-19}}{3 \times 10^8}$

or, $v = \frac{2.55 \times 1.6 \times 10^{-19}}{3 \times 10^8 \times 1.67 \times 10^{-27}} = 0.814$ m/s

2. For hydrogen like atom energy of the nth Bohr orbit

$E_n = -\frac{13.6}{n^2} Z^2$ eV/atom

For transition from $n = 5$ to $n = 4$,

$\frac{h_c}{\lambda} = 13.6 \times 9\left[\frac{1}{16} - \frac{1}{25}\right] = \frac{13.6 \times 9 \times 9}{16 \times 25} = 2.754$ eV

For transition from $n = 4$ to $n = 3$,

$\frac{hc}{\lambda} = 13.6 \times 9\left[\frac{1}{9} - \frac{1}{16}\right] = \frac{13.6 \times 9 \times 7}{9 \times 16} = 5.95$eV

Clearly longer wave length will correspond to transition from $n = 5$ to $n = 4$

For photoelectric effect, $h\nu' - \phi = eV_0$, where ϕ = work function

$5.95 \times 1.6 \times 10^{-19} - W = 1.6 \times 10^{-19} \times 3.95$

$\therefore \phi = 2 \times 1.6 \times 10^{-19} = 2$ eV

Again applying $h\nu - W = eV'_0$

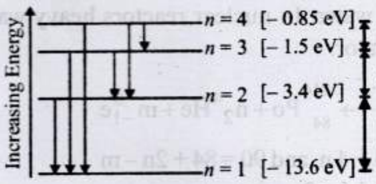
$2.754 \times 1.6 \times 10^{-19} - 2 \times 1.6 \times 10^{-19} = 1.6 \times 10^{-19} V'_0$

$\therefore V'_0 = 0.754$ V (stopping potential for longer wavelength)

3. (i) Since the atoms emit radiation of only six different photon energies

$$\therefore n_f\left(\frac{n_f - 1}{2}\right) = 6 \text{ or, } n_f = 4 \text{ (Final quantum number)}$$

The transition state of six different photon energies as shown in figure.



Since after absorbing monochromatic light, some of the emitted photons have energy more and some have less than 2.7 eV, this indicates that the excited level B is $n = 2$ if $n = 3$ is the excited level then energy less than 2.7 eV is not possible.

(ii) For hydrogen like atoms

$$E_n = \frac{-13.6}{n^2} Z^2 \text{ eV/atom}$$

$$E_4 - E_2 = \frac{-13.6}{16} Z^2 - \left(\frac{-13.6}{4}\right) Z^2 = 2.7$$

$$\Rightarrow Z^2 \times 13.6 \left[\frac{1}{4} - \frac{1}{16}\right] = 2.7 \Rightarrow Z^2 = \frac{2.7}{13.6} \times \frac{4 \times 16}{12}$$

$$\text{Ionisation enrgy, I.E.} = 13.6 Z^2 \left(\frac{1}{1^2} - \frac{1}{\infty^2}\right)$$

$$= 13.6 \times \frac{2.7}{13.6} \times \frac{4 \times 16}{12} = 14.46 \text{ eV}$$

(iii) Maximum energy of the emitted photons

Corresponds to transition from $n = 4$ to $n = 1$

$$E_4 - E_3 = -13.6 Z^2 \left(\frac{1}{4^2} - \frac{1}{1^2}\right)$$

$$= 13.6 \times \frac{2.7}{13.6} \times \frac{4 \times 16}{12} \times \frac{15}{16} = 13.5 \text{ eV}$$

And minimum energy of the emitted photons-corresponds to transition from n = 4 to $n = 3$

$$E_4 - E_3 = -13.6 Z^2 \left(\frac{1}{4^2} - \frac{1}{3^2}\right)$$

$$= 13.6 \times \frac{2.7}{13.6} \times \frac{4 \times 16}{12} \times \frac{7}{9 \times 16} = 0.7 \text{eV}$$

4. (i) According to Bohr for hydrogen/hydrogen like atom,

$$E_n = \frac{-13.6 z^2}{n^2} eV, \; E_2 = -\frac{13.6}{n^2} Z^2, \; E_3 = -\frac{13.6}{9} Z^2$$

$$\therefore E_3 - E_2 = -13.6 Z^2 \left(\frac{1}{9} - \frac{1}{4}\right) = +\frac{13.6 \times 5}{36} Z^2$$

But $E_3 - E_2 = 47.2$ eV (Given)

$$\therefore \frac{13.6 \times 5}{36} Z^2 = 47.2 \quad \therefore Z = \frac{\sqrt{47.2 \times 36}}{13.6 \times 5} = 5$$

(ii) Energy required to excite the electron from the Bohr orbit $n = 3$ to $n = 4$ $E_4 = \dfrac{-13.6}{16} Z^2$

$$\therefore E_4 - E_3 = -13.6 Z^2 \left[\frac{1}{16} - \frac{1}{9}\right] = -13.6 Z^2 \left[\frac{9 - 16}{9 \times 16}\right]$$

$$= \frac{+13.6 \times 25 \times 7}{9 \times 16} = 16.53 \text{eV}$$

(iii) The wavelength of radiation required to remove the electron from $n = 1$ to $n = \infty$ $E_1 = -\dfrac{13.6}{1} \times 25 = -340$ eV

$\therefore E = E_\infty - E_1 = 340 \text{ eV} = 340 \times 1.6 \times 10^{-19}$ J $[E_\infty = 0 \text{ eV}]$

But $E = \dfrac{hc}{\lambda}$

$$\therefore \lambda = \frac{hc}{E} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{340 \times 10^{-19} \times 1.6} = 3.65 \times 10^{-19} \text{m}$$

(iv) Total Energy of 1st orbit

$$= \frac{-13.6 z^2}{n^2} = \frac{13.6 \times 5^2}{1} = -340 \text{ eV}$$

We know that – (T.E). = K.E. [in case of electron revolving around nucleus]

and 2T.E. = P.E.

$\therefore$ K.E. = 340 eV and P.E. = 2 × – 340 = – 680 eV

Angular momentum in 1st orbit : According to Bohr's postulate,

$$\text{Angular moment } L = mvr = \frac{nh}{2\pi}$$

$$\text{For } n = 1, L = \frac{nh}{2\pi} = \frac{6.6 \times 10^{-34}}{2\pi} = 1.05 \times 10^{-34} \text{J-s.}$$

(v) Radius of first Bohr orbit

$$r_1 = \frac{5.3 \times 10^{-11}}{Z} = \frac{5.3 \times 10^{-11}}{5} = 1.06 \times 10^{-11} \text{ m}$$